Trustworthy Machine Learning

Unlearning 1

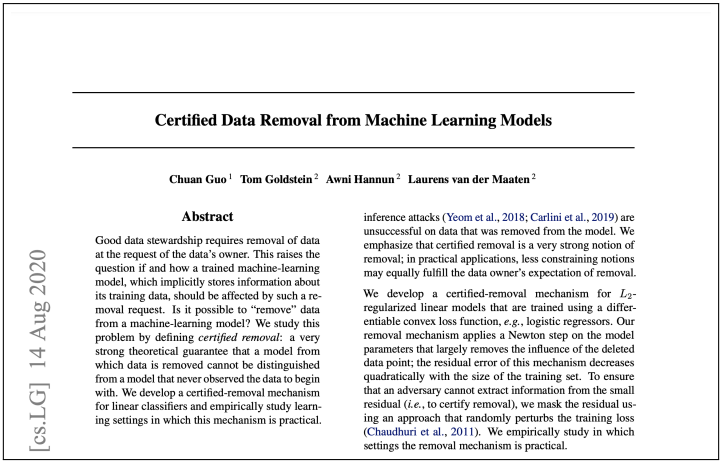
Sangdon Park

POSTECH

Why Unlearning?

- Remove sensitive or private data from a trained model
- Remove data for data-poisoning attacks
- ...

Certified Removal with A Linear Assumption



[cs.LG] 14 Aug 2020

Certified Data Removal from Machine Learning Models

Chuan Guo [1] Tom Goldstein [2] Awni Hannun [2] Laurens van der Maaten [2]

Abstract

Good data stewardship requires removal of data at the request of the data's owner. This raises the question if and how a trained machine-learning model, which implicitly stores information about its training data, should be affected by such a removal request. Is it possible to "remove" data from a machine-learning model? We study this problem by defining *certified removal*: a very strong theoretical guarantee that a model from which data is removed cannot be distinguished from a model that never observed the data to begin with. We develop a certified-removal mechanism for linear classifiers and empirically study learning settings in which this mechanism is practical.

inference attacks (Yeom et al., 2018; Carlini et al., 2019) are unsuccessful on data that was removed from the model. We emphasize that certified removal is a very strong notion of removal; in practical applications, less constraining notions may equally fulfill the data owner's expectation of removal.

We develop a certified-removal mechanism for L_2-regularized linear models that are trained using a differentiable convex loss function, *e.g.*, logistic regressors. Our removal mechanism applies a Newton step on the model parameters that largely removes the influence of the deleted data point; the residual error of this mechanism decreases quadratically with the size of the training set. To ensure that an adversary cannot extract information from the small residual (*i.e.*, to certify removal), we mask the residual using an approach that randomly perturbs the training loss (Chaudhuri et al., 2011). We empirically study in which settings the removal mechanism is practical.

- ICML20
- The term "machine unlearning" seems to firstly appear in Cao and Yang [2015].
- Quite early (and I guess the first) certified removal work.

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   - learning: gradient descent
   - unlearning: gradient ascent
4. Gradient "ascent" almost remove this information, but not perfect.
5. Let's also add noise to hide the remaining information.

Definition: ε-Certified Removal

Given $\varepsilon > 0$, we say that removal mechanism M performs ε-certified removal (ε-CR) for a learning algorithm A if $\forall \mathcal{T} \subseteq \mathcal{H}, \mathcal{D} \subseteq \mathcal{Z}, z \in \mathcal{D}$

$$e^{-\varepsilon} \leq \frac{\mathbb{P}\{M(A(\mathcal{D}), \mathcal{D}, z) \in \mathcal{T}\}}{\mathbb{P}\{A(\mathcal{D} \setminus z) \in \mathcal{T}\}} \leq e^{\varepsilon},$$

where the probability is taken over the randomness of A and M.

- ε: an unlearning parameter.
- $\mathcal{X}$: example space
- $\mathcal{Y}$: label space
- $\mathcal{Z} \coloneqq \mathcal{X} \times \mathcal{Y}$
- $\mathcal{D} \subseteq \mathcal{Z}$: a training set
- $\mathcal{H}$: a hypothesis set
- $A : \mathcal{D} \to \mathcal{H}$: a (randomized) learning algorithm
- In short, we wish to have $M(A(\mathcal{D}), \mathcal{D}, z) \approx A(\mathcal{D} \setminus z)$

Definition: (ε, δ)-Certified Removal

Given $\varepsilon, \delta > 0$, we say that removal mechanism M performs (ε, δ)-certified removal for a learning algorithm A if $\forall \mathcal{T} \subseteq \mathcal{H}, \mathcal{D} \subseteq \mathcal{Z}, z \in \mathcal{D}$

$$\mathbb{P}\{M(A(\mathcal{D}), \mathcal{D}, z) \in \mathcal{T}\} \leq e^{\varepsilon}\mathbb{P}\{A(\mathcal{D} \setminus z) \in \mathcal{T}\} + \delta \quad \text{and}$$
$$\mathbb{P}\{A(\mathcal{D} \setminus z) \in \mathcal{T}\} \leq e^{\varepsilon}\mathbb{P}\{M(A(\mathcal{D}), \mathcal{D}, z) \in \mathcal{T}\} + \delta.$$

- δ upper bounds the failure probability of

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Candidate Method: Exact Removal

Exact Removal

$$M(A(\mathcal{D}), \mathcal{D}, z) \coloneqq A(\mathcal{D} \setminus z)$$

- M is trivially 0-CR.

$$e^0 \leq \frac{\mathbb{P}\{A(\mathcal{D} \setminus z) \in \mathcal{T}\}}{\mathbb{P}\{A(\mathcal{D} \setminus z) \in \mathcal{T}\}} \leq e^0,$$

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- Impractical as we need to retrain a model whenever a training sample is removed.

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- The DP of A is a sufficient condition for certified removal by setting M as an identity function, *i.e.*, $M(A(\mathcal{D}), \mathcal{D}, z) \coloneqq A(\mathcal{D})$ and $\mathcal{D}' \coloneqq \mathcal{D} \setminus z$.

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  - But, DP requires (one-time) retraining and usually introduces poor performance.
- However, the DP of A is not a necessary condition for certified removal.
  - A nearest-neighbor classifier is not differentially private but it can be 0-CR.
- "Retraining from scratch" and "DP" are two extreme removal methods (in removal efficiency).

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- Goal: given a training sample (x, y) to remove, find $\tilde{w}$ such that

$$\tilde{w} \approx A(\tilde{\mathcal{D}}),$$

  where $\tilde{\mathcal{D}} \coloneqq \mathcal{D} \setminus \{(x, y)\}$.

(Not Yet Certified) Removal Mechanism

Newton update removal mechanism M

$$\tilde{w} = M(w^*, \mathcal{D}, (x_n, y_n)) \coloneqq w^* + H_{w^*}^{-1}\Delta$$

- WLOG, remove the last sample (x_n, y_n)
- $w^* \coloneqq \arg\min_w L(w; \mathcal{D}) = A(\mathcal{D})$
- $\tilde{w}^* \coloneqq \arg\min_w L(w; \tilde{\mathcal{D}}) = A(\tilde{\mathcal{D}})$
- $H_{w^*} \coloneqq \nabla^2 L(w^*; \tilde{\mathcal{D}})$: the Hessian of $L(\cdot, \tilde{\mathcal{D}})$ at w^*
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- Similar to the Newton's update, *i.e.,*

$$w_{t+1} = w_t - \left(\nabla^2 \ell(w_t^T x_n, y_n)\right)^{-1} \nabla \ell(w_t^T x_n, y_n)$$

  - The Newton's update: learn (x_n, y_n)
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  - The Newton's update removal mechanism: unlearn (x_n, y_n)
- Why Newton? Fast removal (*i.e.,* removal with one step update)

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Newton update removal mechanism M

$$\tilde{w} = M(w^*, \mathcal{D}, (x_n, y_n)) := w^* + H_{w^*}^{-1}\Delta = w^* + (\nabla^2 L(w^*; \tilde{\mathcal{D}}))^{-1}(\lambda w^* + \nabla \ell((w^*)^T x_n, y_n))$$

- Why?
  - Recall that $w^* := \arg\min_w L(w; \mathcal{D}) = A(\mathcal{D})$
  - Recall that $\tilde{w}^* := \arg\min_w L(w; \tilde{\mathcal{D}}) = A(\tilde{\mathcal{D}})$
  - Due to the Taylor expansion (a.k.a. quadratic approximation),

$$L(w; \tilde{\mathcal{D}}) \approx L(w^*; \tilde{\mathcal{D}}) + \nabla L(w^*; \tilde{\mathcal{D}})(w - w^*)$$

  - We wish to find $\tilde{w}^*$ such that $\nabla L(\tilde{w}^*; \tilde{\mathcal{D}}) = 0$. Thus, the following holds:

$$0 \approx \nabla L(w^*; \tilde{\mathcal{D}}) + \nabla^2 L(w^*; \tilde{\mathcal{D}})(\tilde{w}^* - w^*) \quad \Longrightarrow \quad \tilde{w}^* \approx w^* - (\nabla^2 L(w^*; \tilde{\mathcal{D}}))^{-1} \nabla L(w^*; \tilde{\mathcal{D}})$$

  - Addtionally, we have

$$0 = \nabla L(w^*; \mathcal{D}) = \sum_{i=1}^{n} \nabla \ell((w^*)^T x_i, y_i) + \lambda n w^* = \nabla L(w^*; \tilde{\mathcal{D}}) + \nabla \ell((w^*)^T x_n, y_n) + \lambda w^*$$

    - ★ Additional simplification for efficiency by leveraging recursive relation.

Measuring the Failure of Unlearning

Gradient residual

$$\|\nabla L(\tilde{w}; \tilde{\mathcal{D}})\|_2$$

- In the strongly convex setup, the gradient completely characterizes unlearning.
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- Can we bound this quantity?

Bound on the Failure of Unlearning

Theorem

Suppose that $\|\nabla \ell(w^T x_i, y_i)\|_2 \leq C$ *for any* (x_i, y_i) *and* w, ℓ'' *is* γ*-Lipschitz, and* $\|x_i\|_2 \leq 1$ *for all* x_i. *Then, we have*

$$\|\nabla L(\tilde{w}; \tilde{\mathcal{D}})\|_2 \leq \frac{4\gamma C^2}{\lambda^2 (n-1)}.$$

- Strong-convexity is used here.
- See the paper for the data-dependent bound.

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- As $n \to \infty$, $\|\nabla(\tilde{w}; \tilde{\mathcal{D}})\|_2 \to 0$. Is it enough?
- Claim: the gradient may leak information on the unlearned sample.
  - Consider $\mathcal{D} = \{(e_1, 1), \ldots, (e_d, d)\}$, where e_i for $1 \leq i \leq d$ are the standard basis vectors.
  - The regressor is initialized with zero.
  - $w_i \neq 0$ if (e_i, i) is included in $\mathcal{D}$.
  - An approximate removal will still leave w_i small.
  - $\mathbb{1}(w_i \neq 0)$ indicates the existence of $(e_i, i) \in \mathcal{D}$.

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- Not easy to remove; then how to hide this leaked information?
  - Add noise!

Loss Perturbation

Perturbed empirical risk

$$L_b(w; \mathcal{D}) := \sum_{i=1}^{n} \ell(w^T x_i, y_i) + \frac{\lambda n}{2} \|w\|_2^2 + b^T w$$

- b is randomly drawn from some distribution (*i.e.*, draw b and optimize)
- $b^T w$: Add noise during training time.

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- $b^T w$: Add noise during training time.
  - This masks the information in the gradient residual $\nabla L_b(\tilde{w}; \tilde{\mathcal{D}})$!.

Gradient Residual of Loss Perturbation

Perturbed gradient residual

$$\nabla L_b(w;\tilde{\mathcal{D}}) = \underbrace{\sum_{i=1}^{n-1} \nabla \ell(w^T x_i, y_i) + \lambda(n-1)w}_{\nabla L(w;\tilde{\mathcal{D}})} + b$$

- Recall that we use $\nabla L(\cdot, \tilde{\mathcal{D}})$ to measure the failure of unlearning ($\approx$ information leakage)
- (before) Cannot hide the gradient residual
  - $\tilde{w}^* = \arg\min_w L(w;\tilde{\mathcal{D}})$
  - $\tilde{w}$: the Newton removal mechanism w.r.t. L
  - $\nabla L(\tilde{w}^*;\tilde{\mathcal{D}}) = 0$ but having $\nabla L(\tilde{w};\tilde{\mathcal{D}}) = 0$ is not easy for fast/approximate removal, leaking information.
- (after) Potentially hide the gradient residual
  - $\tilde{w}^* = \arg\min_w L_b(w;\tilde{\mathcal{D}})$
  - $\tilde{w}$: the Newton removal mechanism w.r.t. L_b
  - $\nabla L_b(\tilde{w}^*;\tilde{\mathcal{D}}) = 0$ implies $\nabla L(\tilde{w}^*;\tilde{\mathcal{D}}) = -b$.
  - As $\nabla L(\tilde{w}^*;\tilde{\mathcal{D}})$ is noisy, anyway $\nabla L(\tilde{w};\tilde{\mathcal{D}}) \neq 0$ does not leak information.

(Finally) ε-Certified Removal

Theorem

Let A be the learning algorithm that minimizes $L_b(w; \mathcal{D})$ and M be the Newton update removal mechanism. Suppose that $\|\nabla \ell(w^T x_i, y_i)\|_2 \leq C$ for any (x_i, y_i) and w, ℓ'' is γ-Lipschitz, and $\|x_i\|_2 \leq 1$ for all x_i. If $b_1, b_2 \sim p(b) \propto e^{-\frac{\varepsilon}{\varepsilon'}\|b\|_2}$ (where $\varepsilon' := \frac{4\gamma C^2}{\lambda^2(n-1)}$) and $\|b_1 - b_2\|_2 \leq \varepsilon'$ then M is ε-CR for A.

- ε: a user-specified unlearning parameter.
- $b_1, b_2 \sim p(b) \propto e^{-\frac{\varepsilon}{\varepsilon'}\|b\|_2}$ and $\|b_1 - b_2\|_2 \leq \varepsilon'$: *e.g.*, drawing from a distribution with a constraint (realistic?)
  - A high-probable CR definition in the paper can relax the constraint on the distribution.
- The ideal retraining algorithm: $\tilde{w}^* = \arg\min_w L_b(w, \tilde{\mathcal{D}})$

(Finally) Certified Removal: A Proof Sketch

- Recall the certified removal (CR) definition:

$$\frac{\mathbb{P}\left\{M(A(\mathcal{D}), \mathcal{D}, x) \in \mathcal{T}\right\}}{\mathbb{P}\left\{A(\tilde{\mathcal{D}}) \in \mathcal{T}\right\}} = \frac{\mathbb{P}\{\tilde{w} \in \mathcal{T}\}}{\mathbb{P}\{\tilde{w}^* \in \mathcal{T}\}} \stackrel{?}{\leq} e^{\varepsilon}$$

  Here, the probability is taken over the randomness of the algorithm.

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- The randomness is from the loss perturbation by b. Suppose $b_1, b_2 \sim p(b) \propto e^{-\frac{\varepsilon}{\varepsilon'}\|b\|_2}$. Then,

$$\begin{aligned}\frac{p(b_1)}{p(b_2)}&=\exp\left\{-\frac{\varepsilon}{\varepsilon'}\left(\|b_1\|_2-\|b_2\|_2\right)\right\}=\exp\left\{\frac{\varepsilon}{\varepsilon'}\left(\|b_2\|_2-\|b_1\|_2\right)\right\}\\&\leq\exp\left\{\frac{\varepsilon}{\varepsilon'}\left(\|b_2-b_1\|_2\right)\right\}\leq e^{\varepsilon}\end{aligned}$$

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- From Theorem 2 of the paper,

$$\frac{p(b_1)}{p(b_2)} \leq e^{\varepsilon} \quad \Longrightarrow \quad \frac{\mathbb{P}\{\tilde{w} \in \mathcal{T}\}}{\mathbb{P}\{\tilde{w}^* \in \mathcal{T}\}} \leq e^{\varepsilon}.$$

(ε, δ)-Certified Removal

Theorem

Let A be the learning algorithm that minimizes $L_b(w; \mathcal{D})$ and M be the Newton update removal mechanism. Suppose that $\|\nabla \ell(w^T x_i, y_i)\|_2 \leq C$ for any (x_i, y_i) and w, ℓ'' is γ-Lipschitz, and $\|x_i\|_2 \leq 1$ for all x_i. If $b \sim \mathcal{N}(0, c\frac{\varepsilon'}{\varepsilon})$ with $c > 0$ (where $\varepsilon' := \frac{4\gamma C^2}{\lambda^2(n-1)}$), then M is (ε, δ)-CR for A with $\delta = 1.5e^{-c^2/2}$.

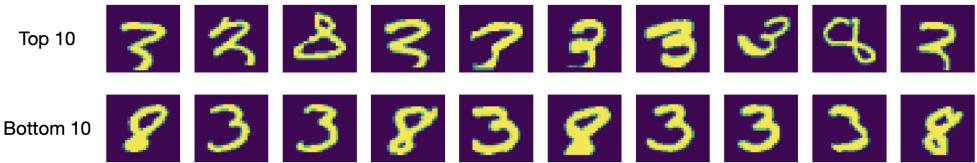
- More complex but more practical

Results: Fast

Dataset	MNIST (§4.1)	LSUN (§4.2)	SST (§4.2)	SVHN (§4.3)
Removal setting	CR Linear	Public Extractor + CR Linear	Public Extractor + CR Linear	DP Extractor + CR Linear
Removal time	0.04s	0.48s	0.07s	0.27s
Training time	15.6s	124s	61.5s	1.5h

- Removal on the last linear layer
- Faster than retraining

Results: Easy v.s. Hard

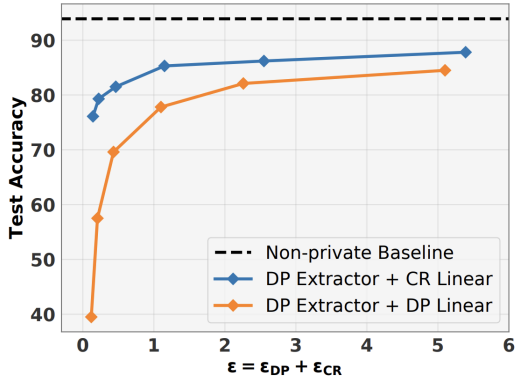


- Recall the Newton update removal:

$$\tilde{w} = M(w^*, \mathcal{D}, (x_n, y_n)) \coloneqq w^* + H_{w^*}^{-1}\Delta$$

- Top 10: 10 examples with higher $\|H_{w^*}^{-1}\Delta\|_2$
- Bottom 10: 10 examples with lower $\|H_{w^*}^{-1}\Delta\|_2$
- Unusual samples are not easy to undo!
  - Removing outliers is harder.
  - The model tends to memorize unusual samples.

Results: CR v.s. DP



- Why not simply use DP?
  - CR Linear (from minimization of perturbed loss) is more accurate than DP Linear; looks better than DP. Why?
- Why not use non-DP extractor?

Conclusion

- How to evaluate the success of unlearning?
- Is the linear assumption critical?
  - We can remove data from the last linear layer of a deep network, which seems to be enough?
- We need to retrain a linear model with noise; not useful?

Reference I

Y. Cao and J. Yang. Towards making systems forget with machine unlearning. In *2015 IEEE symposium on security and privacy*, pages 463–480. IEEE, 2015.